WORK IT OUT

A maths workbook — concepts, worked examples, graded practice and number puzzles

Do the Learn First pages before the practice. Do the practice before the puzzles. Full solutions are at the back — but they only help you after you have tried.

Part	What it is	How to use it	Time
1	Learn First	Read and annotate. Copy every worked example by hand.	25 min
2	Worked Examples	Cover the solution. Try. Then compare line by line.	20 min
3	Guided Practice	Steps are given; you fill the gaps.	15 min
4	Practice Set A · Fluency	Straight application. Aim for speed and accuracy.	20 min
5	Practice Set B · Reasoning	Multi-step. Show every line.	25 min
6	Practice Set C · Challenge	Stretch problems. One may beat you — that is fine.	25 min
7	The Crossnumber	Answers are numbers. Wrong maths will not fit.	20 min
8	Number Search, Magic Square & Concept Bingo	Solo or with a partner.	20 min
9	Spot the Mistake	Audit a worked solution. Find the planted error.	15 min
10	Quick-Fire Round	Matching, True/False, Two Truths and a Lie.	15 min
11	The Escape Challenge	Teams of 4.	25 min

Part 1 · Learn First

Trigonometry stops being an abstract set of ratios the moment you tilt your head to look up at a tower or down at a boat. These pages turn every such situation into a right triangle you already know how to solve.

Line of sight, elevation and depression

Every "height and distance" problem starts by naming three things: the observer's eye, the horizontal through that eye, and the line joining the eye to the object.

The **line of sight** is the line drawn from the eye of an observer to the point on the object being viewed.

The **angle of elevation** of a point is the angle between the line of sight and the horizontal, measured when the point is *above* the horizontal level — i.e., when we raise our head to look at it.

The **angle of depression** of a point is the angle between the line of sight and the horizontal, measured when the point is *below* the horizontal level — i.e., when we lower our head to look at it.

Careful: elevation and depression are always measured from the **horizontal**, never from the vertical, and never from the ground unless the observer happens to be standing on it.

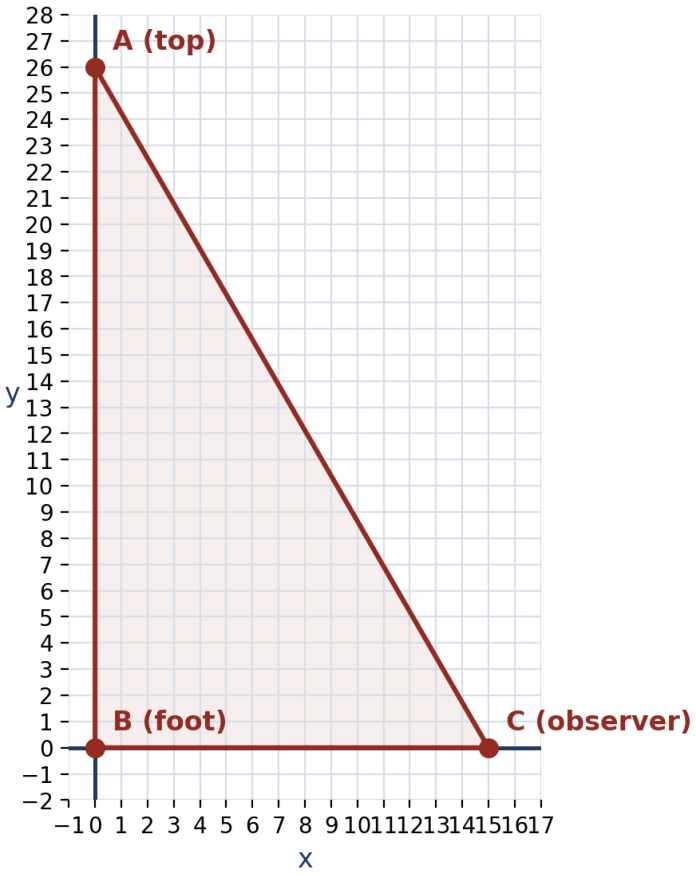

Fig. 9.1 — the line of sight AC makes the angle of elevation with the horizontal BC

Rule to remember

- If the object is **above** eye level, the angle you use is an angle of **elevation**.
- If the object is **below** eye level, the angle you use is an angle of **depression**.
- The angle of elevation of A from B equals the angle of depression of B from A — they are alternate angles on the same transversal line of sight.

The general method for height-and-distance problems

Almost every problem in this chapter reduces to the same four moves.

1. **Draw a simple diagram.** Mark the vertical object, the horizontal ground, and the line of sight. Right angles nearly always appear where a vertical object meets the horizontal ground.
2. **Identify the right triangle** that contains the angle you are given and the side you want.
3. **Choose the correct trigonometric ratio.** Ask: of the sides I know and the side I want, which ratio (sin, cos, tan or their reciprocals) connects them to the given angle?
4. **Solve the equation** for the unknown side, substituting standard values of sin 30°, cos 30°, tan 30°, sin 45°, tan 60°, etc.

Angle	sin	cos	tan
30°	1/2	√3/2	1/√3
45°	1/√2	1/√2	1
60°	√3/2	1/2	√3

> ### Choosing the ratio
>
> - Known **adjacent**, want **opposite** (or vice versa) → use **tan** or **cot**.
> - Known **opposite**, want **hypotenuse** (or vice versa) → use **sin** or **cosec**.
> - Known **adjacent**, want **hypotenuse** (or vice versa) → use **cos** or **sec**.

A worked setup — finding the height of a minar

Suppose a student, standing at eye-height AE from the ground, looks up at the top D of a minar with angle of elevation ∠BAC (Fig. 9.1 style, redrawn as the triangle above).

We would need to know: (i) the distance DE the student stands from the foot of the minar, (ii) the angle of elevation ∠BAC, and (iii) the height AE of the student's eye.

Since $CD = CB + BD$ and $BD = AE$ (the student's height), we only need to find BC. In right triangle ABC, BC is opposite $\angle A$ and AB is adjacent to it, so $\tan A = \frac{BC}{AB}$ gives BC directly.

Finally, add AE to BC to get the full height CD of the minar. This add-the-observer's-height step is easy to forget, and examiners test it often.

Setting up before solving

A student 1.5 m tall stands 20 m from a tower and sees its top at an elevation of 30°. Set up the equation for the tower's height (do not solve yet).

Let the tower be AB, the horizontal from the student's eye meet the tower at E, so AE is the part of the tower above eye level.

In right triangle formed at E: $\tan 30° = \frac{AE}{20}$.

Once AE is found, the full height is $AB = AE + 1.5$ (the student's eye height).

The point: always separate "the part of the object above eye level" from "the observer's own height" — they are added only at the very end.

Part 2 · Worked Examples

Cover the right-hand column with a sheet of paper. Try the problem yourself first. Then uncover it one line at a time and check where your reasoning parted from the model.

Example 1 — A tower stands vertically on the ground. From a point 15 m from the foot of the tower, the angle of elevation of the top is 60°. Find the height of the tower.

Step 1. Let AB be the tower and C the point on the ground, with $CB = 15$ m and $\angle ACB = 60^\circ$.

Step 2. Triangle ACB is right-angled at B. We know the side adjacent to the angle (CB) and want the side opposite (AB), so we use tan.

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{15}$$

Answer: $AB = 15\sqrt{3}$ m ≈ 25.98 m

> **Why this works**
>
> The angle 60° with adjacent side 15 immediately calls for tan 60° = √3 — no need to find the hypotenuse at all.

Example 2 — An electrician must reach a point 1.3 m below the top of a 5 m pole. A ladder inclined at 60° to the horizontal is used. Find the ladder's length and its distance from the pole.

Step 1. The point to reach is B on pole AD, with $BD = AD - AB = 5 - 1.3 = 3.7$ m.

Step 2. BC (the ladder) is the hypotenuse of right triangle BDC. We know the side opposite the 60° angle (BD), and want the hypotenuse, so use sin.

$$\sin 60^\circ = \frac{BD}{BC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3.7}{BC}$$

$$BC = \frac{3.7 \times 2}{\sqrt{3}} \approx 4.28 \text{ m}$$

Step 3. For the foot distance DC, we know opposite and want adjacent, so use cot.

$$\cot 60^\circ = \frac{DC}{BD} = \frac{1}{\sqrt{3}} \Rightarrow DC = \frac{3.7}{\sqrt{3}} \approx 2.14 \text{ m}$$

> **Why this works**
>
> Two different ratios were needed for two different unknowns in the *same* triangle — always ask "which two sides does this question involve?" before picking sin, cos or tan.

Example 3 — From a point P on the ground, a 10 m building's top has elevation 30°, and a flagstaff on top of it has elevation 45° from the same point. Find the flagstaff's length and the distance of the building from P.

Step 1. Let $AB = 10$ m be the building, BD the flagstaff, and P the point. Two right triangles share the base PA: triangle PAB and triangle PAD.

Step 2. In right triangle PAB: $\tan 30^\circ = \frac{AB}{AP} = \frac{10}{AP}$, so $\frac{1}{\sqrt{3}} = \frac{10}{AP}$, giving $AP = 10\sqrt{3} \approx 17.32$ m.

Step 3. Let $DB = x$, so $AD = 10 + x$. In right triangle PAD: $\tan 45^\circ = \frac{AD}{AP} = \frac{10+x}{10\sqrt{3}}$.

Step 4. Since $\tan 45^\circ = 1$: $10 + x = 10\sqrt{3}$, so $x = 10(\sqrt{3} - 1) \approx 7.32$ m.

> **Why this works**
>
> The base AP found from the first triangle is reused, unchanged, in the second triangle — that reuse is what makes two-triangle problems solvable at all.

Example 4 — The shadow of a tower is 40 m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.

Step 1. Let the tower be $AB = h$ m. Let $BC = x$ m be the shadow at 60°, so the shadow at 30° is $BD = x + 40$ m.

Step 2. In right triangle ABC: $\tan 60^\circ = \frac{h}{x}$, so $h = x\sqrt{3}$. (1)

Step 3. In right triangle ABD: $\tan 30^\circ = \frac{h}{x+40}$, so $\frac{1}{\sqrt{3}} = \frac{h}{x+40}$, i.e. $h\sqrt{3} = x + 40$. (2)

Step 4. Substitute (1) into (2): $x\sqrt{3} \cdot \sqrt{3} = x + 40 \Rightarrow 3x = x + 40 \Rightarrow x = 20$.

Answer: $h = 20\sqrt{3}$ m ≈ 34.64 m

> **Why this works**
>
> Two unknowns, two right triangles sharing the same height h — this "two elevations, one tower" pattern reappears throughout the exercise (poles, buildings, TV towers).

Part 3 · Guided Practice

The method is laid out for you. Fill in the blanks. If you can do these without looking back at Part 2, you are ready for Part 4.

1. The angle of elevation of the top of a tower from a point 30 m from its foot is 30°. Find the height of the tower.

The known side (30 m) is _____ (adjacent/opposite) to the 30° angle.

The unknown height is _____ (adjacent/opposite) to the 30° angle, so use _____ .

$\tan 30^\circ = \frac{h}{30}$, so $h = 30 \times$ _____ = _____ m.

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2. A kite flies at a height of 60 m. The string, tied to the ground, is inclined at 60° to the horizontal. Find the length of the string (no slack).

The height 60 m is the side _____ (opposite/adjacent) the 60° angle.

The string is the _____ (hypotenuse/adjacent side) of the right triangle.

$\sin 60^\circ = \frac{60}{L}$, so $L = \frac{60}{\sin 60^\circ} =$ ¿ _____ m.

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3. A 1.5 m tall boy stands away from a 30 m building. The elevation of the top increases from 30° to 60° as he walks towards it. Find the distance he walked.

First subtract the boy's height from the building: the part above his eye level is $30 - 1.5 =$ ¿ _____ m.

At 30°: $\tan 30^\circ = \frac{28.5}{d_1}$, so $d_1 =$ ¿ _____ m.

At 60°: $\tan 60^\circ = \frac{28.5}{d_2}$, so $d_2 =$ ¿ _____ m.

Distance walked ¿ $d_1 - d_2 =$ ¿ _____ m.

Practice Set A · Fluency

Straight application of the method. Aim to get every one right, then aim to get them right quickly.

#	Question	Your working and answer
1	A circus artist climbs a 20 m rope tied from the top of a vertical pole to the ground, making an angle of 30° with the ground. Find the height of the pole.	
2	A tree breaks in a storm; the broken top touches the ground 8 m from the foot, making an angle of 30° with the ground. Find the height of the tree.	
3	A slide for young children has its top at height 1.5 m, inclined at 30° to the ground. Find the length of the slide.	
4	The angle of elevation of the top of a tower from a point 30 m from its foot is 30°. Find the height of the tower.	
5	A kite flies at a height of 60 m; its string is inclined at 60° to the ground with no slack. Find the length of the string.	
6	From a point on the ground, the angle of elevation of the top of a 10 m tall pole is 45°. How far is the point from the foot of the pole?	
7	A ladder 10 m long rests against a wall, inclined at 60° to the ground. How high up the wall does it reach?	

Practice Set B · Reasoning

Two or more steps, or a step you have to choose for yourself. Show every line — marks live in the working.

#	Question	Your working and answer
1	A 1.5 m tall boy stands away from a 30 m building; the elevation of the top increases from 30° to 60° as he walks towards it. Find the distance he walked.	
2	From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed on top of a 20 m building are 45° and 60° respectively. Find the height of the tower.	
3	A statue 1.6 m tall stands on a pedestal. From a point on the ground, the top of the statue has elevation 60° and the top of the pedestal has elevation 45°. Find the height of the pedestal.	
4	The angle of elevation of the top of a building from the foot of a tower is 30°, and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.	
5	Two poles of equal height stand on either side of a road 80 m wide. From a point between them, the elevations of the tops are 60° and 30°. Find the height of the poles and the distances of the point from each pole.	
6	From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.	

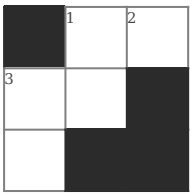
Practice Set C · Challenge

Harder, and deliberately so. If one defeats you, write down exactly where you got stuck. That sentence is worth more than a copied answer.

#	Question	Your working and answer
1	A TV tower stands on a canal bank. From a point directly opposite on the other bank, the elevation of the top is 60°. From another point 20 m further back on the same line, the elevation is 30°. Find the height of the tower and the width of the canal.	
2	As observed from the top of a 75 m lighthouse, the angles of depression of two ships in line on the same side are 30° and 45°. Find the distance between the ships.	
3	A 1.2 m tall girl spots a balloon moving horizontally at a height of 88.2 m; its elevation from her eyes reduces from 60° to 30° over some time. Find the distance travelled by the balloon.	
4	A man at the top of a tower observes a car on a straight highway leading to its foot at an angle of depression of 30°; six seconds later the angle of depression is 60°. Find the further time the car takes to reach the foot of the tower.	
5	The angles of depression of the top and bottom of an 8 m building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the multi-storeyed building and the distance between the two buildings.	

Part 7 · The Crossnumber

Every answer is a number. One digit per square. Because answers cross, a wrong calculation simply will not fit — this puzzle marks itself.



ACROSS

1. Distance in metres from a 15√3 m tower where the elevation is 60° (the base itself) *(2 digits)*

3. Shadow length x in metres solved in the 30°/60°/40 m-longer-shadow tower problem *(2 digits)*

DOWN

1. Height of a pole whose 20 m rope-ladder meets the ground at 30° (tan 30° gives the base, sin gives this) *(2 digits)*

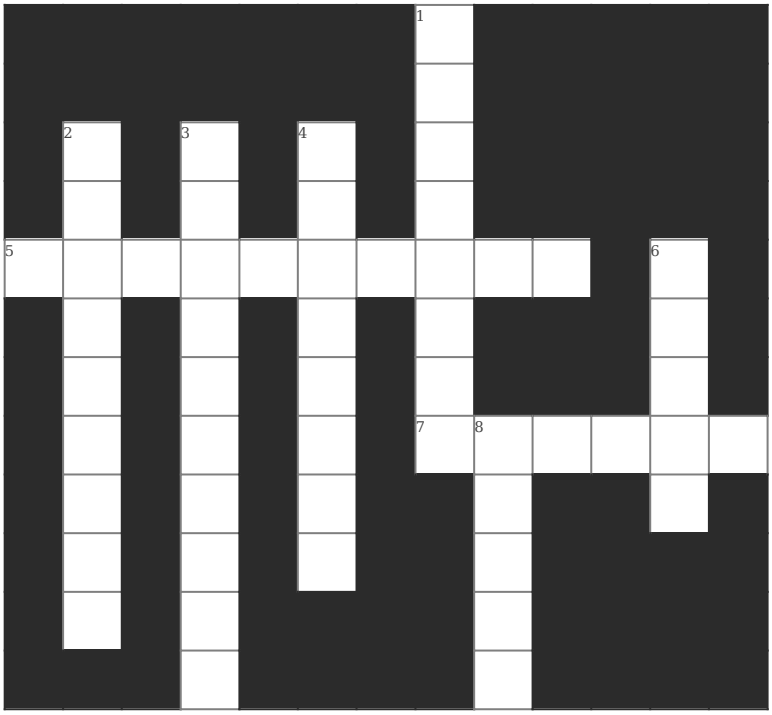
2. Height in metres a pole rises if its 10 m ladder meets the ground at 30° (half the ladder) *(1 digits)*

3. Approximate height in metres of a tower 15 m base with 60° elevation, rounded (15√3 ≈ 25.98) *(2 digits)*

Bonus questions that did not fit the grid — work them out anyway: Height in metres of a tower whose 30 m base gives a 30° elevation reading of exactly this many metres; Height in metres reached by a string flying a kite tied at 60° elevation over a 20√3 m base; Height in metres of a wall reached by a 24 m ladder inclined at 30° (half the ladder length); Height in metres of a broken tree top touching ground 8√3 m away at 30° (adjacent side given); Distance in metres of a building from point P, rounding 10√3 to the nearest whole number

Part 7b · The Crossword

Same rules as an ordinary crossword — but every word is maths vocabulary from this chapter.



ACROSS

5. angle formed with the horizontal when looking down at an object (10)

7. inclined object whose length is usually the hypotenuse (6)

DOWN

1. the direction in which a tower or pole stands (8)

2. angle formed with the horizontal when looking up at an object (9)

3. the reference level from which elevation and depression are measured (10)

4. the person doing the looking (8)

6. a tall vertical structure often used in these word problems (5)

8. measured in degrees between two rays (5)

Part 8 · Number Search & Magic Square

Work out each answer below, then find it hidden in the digit grid — across, down, diagonally or backwards.

1. Distance in metres of the point from a 15√3 m tower (Example 1)
2. Common angle of elevation in degrees seen throughout the exercise
3. Angle of elevation, in degrees, that gives tan = 1
4. Angle of elevation, in degrees, that gives tan = √3
5. Height in metres of a broken tree top touching the ground at 30° (Q2 base)
6. Length in metres of the flying kite's ground-tied string scenario (2×20)
7. Length in metres of the circus artist's rope divided by 2
8. Height in metres of the lighthouse in the two-ships problem

7	3	0	8	5	5	9	5	8	9	3	3	0
2	5	1	5	9	6	1	8	8	4	1	3	5
3	7	1	6	0	9	5	5	4	3	8	4	5
8	9	0	1	7	4	9	6	4	8	3	9	1
3	6	7	4	9	8	2	7	4	0	2	8	1
1	4	8	9	8	7	6	5	0	5	0	9	4
9	8	7	5	0	9	3	3	5	6	6	2	2
7	3	2	1	2	9	5	8	0	3	6	1	4
7	0	1	6	5	7	1	5	0	4	0	2	2
8	7	3	7	8	7	9	9	0	1	2	2	8
5	0	9	3	0	6	1	0	1	0	4	0	8
4	3	9	8	2	8	9	4	4	9	9	1	9
7	5	8	0	1	6	2	0	6	5	4	9	7

☐ 15	☐ 30	☐ 45	☐ 60	☐ 20
☐ 40	☐ 10	☐ 75		

Magic Square — every row, column and diagonal adds to the same total.

	1	**6**
	5	

4		**2**

Magic constant: 15. Fill the empty cells.

Concept Bingo

Fill the empty grid below with any 16 terms from the bank — in any order, one per box. Then work down the clue list: each clue describes one term without naming it. Cross off a box only when you are sure which term its clue is describing. Get four crossed in a row, column or diagonal and you have bingo.

Term bank — choose any 16:

Line of sight	Angle of elevation	Angle of depression	Horizontal level
Right triangle	Opposite side	Adjacent side	Hypotenuse
tan ratio	sin ratio	cos ratio	Tower
Ladder	Shadow	Observer's eye height	Alternate angles
Kite string			

Your bingo card:

Clue list — read each clue and find the term it describes:

① The line drawn from the observer's eye to the point on the object viewed.
② The angle with the horizontal formed when looking up at an object.
③ The angle with the horizontal formed when looking down at an object.
④ The reference line from which elevation and depression are measured.
⑤ The shape formed by a vertical object, the horizontal ground, and the line of sight.
⑥ The side of a right triangle facing the angle in question, away from it.
⑦ The side of a right triangle that lies along one arm of the angle in question.
⑧ The longest side of a right triangle, opposite the right angle.
⑨ The ratio to use when you know the adjacent side and want the opposite side.
⑩ The ratio to use when you know the hypotenuse and want the opposite side.

- The ratio to use when you know the hypotenuse and want the adjacent side.
- A tall vertical structure whose height is often the unknown in these problems.
- An inclined object whose length is usually the hypotenuse of a right triangle.
- The dark patch cast on the ground, whose length changes with the Sun's altitude.
- The measurement that must sometimes be added back once a triangle is solved.
- The pair of equal angles formed when a transversal crosses two parallel horizontals.
- The line, usually treated as straight, along which a kite's height is measured.

Part 9 · Spot the Mistake

Each solution below reaches a wrong answer, and exactly one step is to blame. Find the first wrong step and rewrite it correctly. Marking other people's work is the fastest way to stop making their mistakes.

Spot the Mistake

Elevation or depression?

Step 1. A person on a cliff looks down at a boat; the angle formed with the horizontal is called the angle of elevation.

Step 2. The angle of elevation of the cliff-top from the boat equals this same angle, since they are alternate angles on the same line of sight.

Step 3. So both angles in this situation are called angles of elevation.

_____ The first wrong step is Step _____ . What should it say?

_____

Spot the Mistake

Choosing the ratio for a ladder problem

Step 1. A ladder leans against a wall at 60° to the ground; we know the base distance and want the height reached, so we should use tan 60°.

Step 2. $\tan 60\,° = \frac{height}{base}$, so height ¿ $base \times \tan 60\,°$.

Step 3. If instead we know the ladder's length (hypotenuse) and want the height, we should still use tan.

_____ The first wrong step is Step _____ . What should it say?

_____

Spot the Mistake

Adding the observer's height

Step 1. A 1.5 m tall boy sees a balloon at elevation 30°; the horizontal distance found from $\tan 30\,°$ is the height of the balloon above the ground.

Step 2. The height found from $\tan 30\,°$ is only the height of the balloon **above the boy's eye level**.

Step 3. To get the balloon's height above the ground, we must add the boy's eye height (1.5 m) to that result.

_____ The first wrong step is Step _____ . What should it say?

_____

Part 10 · Quick-Fire Round

Match each term to its correct meaning

Draw a line from each item on the left to its match on the right.

#	Left	Right (shuffled)
1	Line of sight	√3
2	Angle of elevation	The line from the observer's eye to the point being viewed
3	Angle of depression	Angle with the horizontal when looking down at an object
4	Horizontal level	Angle with the horizontal when looking up at an object
5	tan 60°	1/√2
6	sin 45°	The reference line from which elevation and depression are measured

True or False Sprint

Read each statement to yourself (or take turns with a partner). Decide True or False before you check the solutions.

#	Statement	Your call
1	The angle of elevation is always measured from the vertical.	☐ True ☐ False
2	The line of sight is the line from the observer's eye to the object viewed.	☐ True ☐ False
3	If you look down at an object, the angle formed is called the angle of depression.	☐ True ☐ False
4	The angle of elevation of a point from an observer equals the angle of depression of the observer from that point.	☐ True ☐ False
5	tan 30° = √3.	☐ True ☐ False
6	When the base of a right triangle is known and the height is wanted, sin is the correct ratio to use.	☐ True ☐ False
7	An observer's own height must sometimes be added to a trigonometric result to get the full height of an object.	☐ True ☐ False
8	The height of a tower can be found without knowing any angle, using distance alone.	☐ True ☐ False

Two Truths and a Lie

Two statements are true, one is false. Circle the lie — then correct it.

Set 1

(a) The angle of elevation is measured from the horizontal.

(b) The angle of depression is measured from the vertical.

(c) Both angles are measured from a horizontal reference line.

The lie is _____ . Corrected: _____

Set 2

(a) tan 45° = 1.

(b) sin 60° = √3/2.

(c) cos 30° = 1/2.

The lie is _____ . Corrected: _____

Set 3

(a) In a ladder problem, the ladder is usually the hypotenuse.

(b) The angle of elevation from the boat equals the angle of depression from the cliff.

(c) A taller observer always sees a larger angle of elevation to the same point.

The lie is _____ . Corrected: _____

Part 11 · The Escape Challenge

Five gates, each a calculation. Each gives you one digit. Collect all five to unlock the exit code.

🔒 GATE 1 — The Tower Gate

A tower's shadow is 20 m long when the Sun's elevation is 45°. Find the height of the tower in metres.

Digit: ☐

🔒 GATE 2 — The Ladder Gate

A 10 m ladder leans against a wall at 30° to the ground. Find the height it reaches on the wall (take the value of $10 \times \sin 30°$).

Digit: ☐

🔒 GATE 3 — The Kite Gate

A kite string is 40 m long and inclined at 30° to the ground. Find the height of the kite above the ground (take $40 \times \sin 30°$).

Digit: ☐

🔒 GATE 4 — The Pole Gate

A pole casts no shadow issue here: at elevation 60° from a point 10 m away, the height is $10\sqrt{3} \approx 17.3$ m. Round to the nearest whole metre.

Digit: ☐

🔒 GATE 5 — The Building Gate

A building 30 m from an observer is viewed at elevation 30°. Find its height using $30 \times \tan 30° = 30/\sqrt{3} = 10\sqrt{3} \approx 17.3$; round down to the nearest whole metre.

Digit: ☐

EXIT CODE: ☐ ☐ ☐ ☐ ☐

Formula Sheet

Cut this page out and keep it. Everything in this chapter, on one page.

Angle of elevation

$$Angle between the line of sight and the horizontal, looking up$$

Angle of depression

$$Angle between the line of sight and the horizontal, looking down$$

tan of angle

$$\tan\theta = \frac{opposite}{adjacent}$$

sin of angle

$$\sin\theta = \frac{opposite}{hypotenuse}$$

cos of angle

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

Standard values

$$\tan 30° = \frac{1}{\sqrt{3}}, \tan 45° = 1, \tan 60° = \sqrt{3}$$

Standard values

$$\sin 30° = \frac{12}{,} \sin 45° = \frac{1}{\sqrt{2}}, \sin 60° = \frac{\sqrt{3}}{2}$$

Full Solutions

Not just answers — working. Compare your method with this one; if they differ but both reach the right answer, make sure you understand why yours also works.

Part 3 · Guided Practice

1. adjacent; opposite, tan; h = 30/√3 = 10√3 m · **2.** opposite; hypotenuse; L = 60 × 2/√3 = 40√3 m · **3.** 28.5 m; d1 = 28.5√3 m; d2 = 28.5/√3 = 9.5√3 m; walked = 28.5√3 − 9.5√3 = 19√3 m

Practice Set A

1. $\sin 30^\circ = h/20$, so $h = 20 \times \frac{12}{} = 10$ m.

2. $\tan 30^\circ = h/8$, so $h_{stump} = 8/\sqrt{3}$; broken part (hypotenuse) $= 8/\cos 30^\circ = 16/\sqrt{3}$; total height $= 8/\sqrt{3} + 16/\sqrt{3} = 24/\sqrt{3} = 8\sqrt{3}$ m.

3. $\sin 30^\circ = 1.5/L$, so $L = 1.5/0.5 = 3$ m.

4. $\tan 30^\circ = h/30$, so $h = 30/\sqrt{3} = 10\sqrt{3} \approx 17.32$ m.

5. $\sin 60^\circ = 60/L$, so $L = 60/(\sqrt{3}/2) = 40\sqrt{3} \approx 69.28$ m.

6. $\tan 45^\circ = 10/d$, so $d = 10$ m (since tan 45° = 1).

7. $\sin 60^\circ = h/10$, so $h = 10 \times \sqrt{3}/2 = 5\sqrt{3} \approx 8.66$ m.

Practice Set B

1. Part above eye level $= 30 - 1.5 = 28.5$ m. $d_1 = 28.5\sqrt{3}$ (at 30°), $d_2 = 28.5/\sqrt{3}$ (at 60°). Distance walked $= d_1 - d_2 = 28.5\sqrt{3} - 28.5/\sqrt{3} = 19\sqrt{3} \approx 32.9$ m.

2. In right triangle to building foot: $\tan 45^\circ = 20/AP$, so $AP = 20$ m. To tower top: $\tan 60^\circ = (20 + x)/20$, so $20 + x = 20\sqrt{3}$, $x = 20(\sqrt{3} - 1) \approx 14.64$ m.

3. $\tan 45^\circ = h_{ped}/d$ and $\tan 60^\circ = (h_{ped} + 1.6)/d$. Dividing and solving gives $h_{ped} = 1.6/(\sqrt{3} - 1) = 0.8(\sqrt{3} + 1) \approx 2.19$ m.

4. $\tan 60^\circ = 50/d$, so $d = 50/\sqrt{3}$. $\tan 30^\circ = h/d = h\sqrt{3}/50$; with $\tan 30^\circ = 1/\sqrt{3}$: $h = 50/3 \approx 16.67$ m.

5. Let poles have height h; distances x and $80 - x$. $\tan 60^\circ = h/x$, $\tan 30^\circ = h/(80 - x)$. Solving: $x = 20$ m, $80 - x = 60$ m, $h = 20\sqrt{3} \approx 34.64$ m.

6. $\tan 45^\circ = d_{foot}/7$ gives $d = 7$ m to the building's base-line; $\tan 60^\circ = (7 + t)/7$, so $t = 7\sqrt{3} - 7 = 7(\sqrt{3} - 1) \approx 5.12$ m.

Practice Set C

1. $\tan 60^\circ = h/d$, $\tan 30^\circ = h/(d+20)$. From first, $h = d\sqrt{3}$; substituting: $d\sqrt{3}\cdot\sqrt{3} = d+20 \Rightarrow 3d = d+20 \Rightarrow d = 10$, $h = 10\sqrt{3} \approx 17.3$ m.

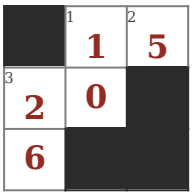
2. $\tan 45^\circ = 75/d_1 \Rightarrow d_1 = 75$; $\tan 30^\circ = 75/d_2 \Rightarrow d_2 = 75\sqrt{3}$. Distance between ships $\downarrow d_2 - d_1 = 75\sqrt{3} - 75 = 75(\sqrt{3}-1) \approx 54.9$ m.

3. Height above eyes $\downarrow 88.2 - 1.2 = 87$ m. $d_1 = 87/\sqrt{3}$ (60°), $d_2 = 87\sqrt{3}$ (30°). Distance $\downarrow d_2 - d_1 = 87\sqrt{3} - 87/\sqrt{3} = 58\sqrt{3} \approx 100.5$ m.

4. $\tan 30^\circ = h/d_1$, $\tan 60^\circ = h/d_2$, and the car covers $d_1 - d_2$ in 6 s. Since $d_1 = h\sqrt{3}$, $d_2 = h/\sqrt{3}$, $d_1 - d_2 = 6v$ and $d_2 = t\cdot v$; solving gives further time $t = 3$ s.

5. With $PD = 8/(\sqrt{3}-1) = 4(\sqrt{3}+1)$: multi-storeyed height $\downarrow PD + 8 = 4(\sqrt{3}+1) + 8 = 4\sqrt{3} + 12 \approx 18.9$ m; distance between buildings $\downarrow 4(\sqrt{3}+1) \approx 10.9$ m.

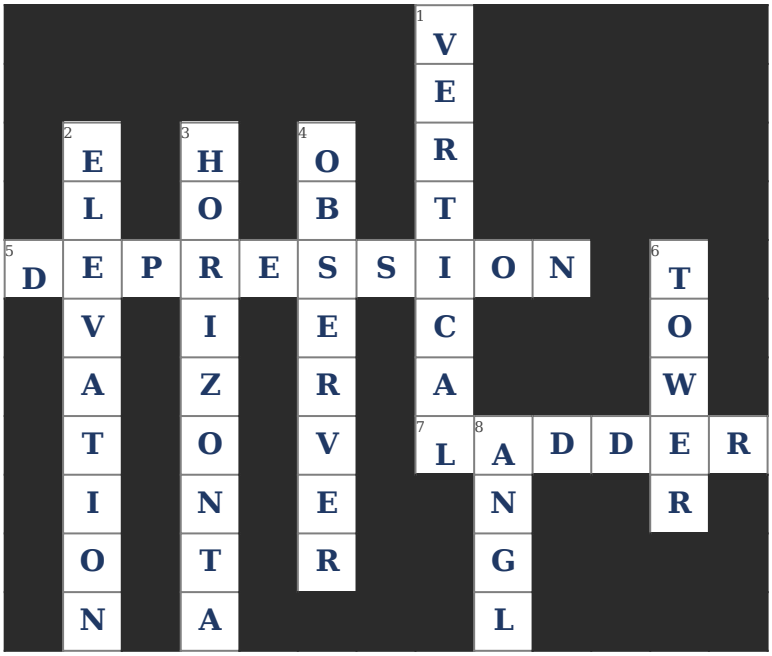
Part 7 · Crossnumber — Solved Grid



Across: 1 = 15 · 3 = 20

Down: 1 = 10 · 2 = 5 · 3 = 26

Part 7b · Crossword — Solved Grid



				L				E				

Across: 5 = DEPRESSION · 7 = LADDER

Down: 1 = VERTICAL · 2 = ELEVATION · 3 = HORIZONTAL · 4 = OBSERVER · 6 = TOWER · 8 = ANGLE

Part 8 · Number Search — Solved Grid

7	3	0	8	5	5	9	5	8	9	3	3	0
2	5	1	5	9	6	**1**	8	8	4	1	3	5
3	7	1	6	**0**	9	5	**5**	4	3	8	4	5
8	9	0	**1**	7	4	9	6	4	8	3	9	1
3	6	7	4	9	8	2	7	**4**	0	2	8	1
1	4	8	9	8	7	6	5	0	**5**	**0**	9	4
9	8	7	5	0	9	3	3	5	**6**	6	2	2
7	3	2	1	2	9	5	8	**0**	**3**	6	1	4
7	0	1	6	**5**	**7**	1	5	0	4	0	2	2
8	7	3	7	8	7	9	9	0	1	2	2	8
5	0	9	3	0	6	1	0	1	**0**	4	0	8
4	3	9	8	2	8	9	4	**4**	9	9	1	9
7	5	8	0	1	6	**2**	**0**	6	5	4	9	7

Part 8 · Magic Square — Solved (constant 15)

8	**1**	**6**
3	**5**	**7**
4	**9**	**2**

Concept Bingo — Clue Key

#	Clue	Term
1	The line drawn from the observer's eye to the point on the object viewed.	Line of sight

#	Clue	Term
2	The angle with the horizontal formed when looking up at an object.	Angle of elevation
3	The angle with the horizontal formed when looking down at an object.	Angle of depression
4	The reference line from which elevation and depression are measured.	Horizontal level
5	The shape formed by a vertical object, the horizontal ground, and the line of sight.	Right triangle
6	The side of a right triangle facing the angle in question, away from it.	Opposite side
7	The side of a right triangle that lies along one arm of the angle in question.	Adjacent side
8	The longest side of a right triangle, opposite the right angle.	Hypotenuse
9	The ratio to use when you know the adjacent side and want the opposite side.	tan ratio
10	The ratio to use when you know the hypotenuse and want the opposite side.	sin ratio
11	The ratio to use when you know the hypotenuse and want the adjacent side.	cos ratio
12	A tall vertical structure whose height is often the unknown in these problems.	Tower
13	An inclined object whose length is usually the hypotenuse of a right triangle.	Ladder
14	The dark patch cast on the ground, whose length changes with the Sun's altitude.	Shadow
15	The measurement that must sometimes be added back once a triangle is solved.	Observer's eye height
16	The pair of equal angles formed when a transversal crosses two parallel horizontals.	Alternate angles
17	The line, usually treated as straight, along which a kite's height is measured.	Kite string

Part 9 · Spot the Mistake

Problem	First wrong step	It should say
Elevation or depression?	Step 2	When you look **down** at an object, the angle formed with the horizontal is the angle of **depression**, not elevation. The boat looking up at the cliff-top sees an angle of elevation — the two angles are equal in value but have different names because they are measured from different observers.
Choosing the ratio for a ladder problem	Step 1	When the **hypotenuse** is known and the **opposite** side is wanted, tan is the wrong ratio because tan never involves the hypotenuse — **sin** must be used instead: $\sin 60° = \frac{height}{ladder\,length}$.
Adding the observer's height	Step 2	A very common slip: the right triangle formed by the line of sight only accounts for the part of the object **above eye level** — the observer's own height must always be added back on at the end.

Part 10 · Matching

#	Left	Correct match
1	Line of sight	The line from the observer's eye to the point being viewed
2	Angle of elevation	Angle with the horizontal when looking up at an object
3	Angle of depression	Angle with the horizontal when looking down at an object
4	Horizontal level	The reference line from which elevation and depression are measured
5	tan 60°	√3
6	sin 45°	1/√2

Part 10 · True or False

1-F · 2-T · 3-T · 4-T · 5-F · 6-F · 7-T · 8-F

Part 10 · Two Truths and a Lie

Set	The lie	Correction
1	b	The angle of depression, like elevation, is measured from the horizontal, not the vertical.
2	c	cos 30° = √3/2, not 1/2 — that is the value of sin 30° (or cos 60°).
3	c	The angle of elevation depends on the horizontal distance and the height above eye level, not on the observer's height alone — a taller observer can see either a larger or smaller angle depending on position.

Part 11 · The Escape Challenge

EXIT CODE: 20 · 5 · 20 · 17 · 17

(Answer: 20) (Answer: 5) (Answer: 20) (Answer: 17) (Answer: 17)

Every skyline, kite string and ladder around you is secretly a right triangle waiting for its angle. You now know exactly which ratio to reach for. □□